Robust Estimation of 3D Human Poses from Single Images Supplementary material

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Abstract

This document contains additional details of using Alternating Direction Method (ADM) to solve (i) the pose estimation problem (Section 1), and (ii) the camera estimation problem (Section 2).

1. 3D Pose Estimation

Given the camera parameters M and the 2D pose x, we estimate the 3D pose by solving the following L_1 minimization problem using ADM:

$$\begin{aligned} \min_{\alpha} \quad & \|x - M(B\alpha + \mu)\|_1 + \theta \|\alpha\|_1 \\ \text{s.t.} \quad & \|C_i (B\alpha + \mu)\|_2^2 = L_i, i = 1, \cdots, t \end{aligned} \tag{1}$$

We introduce two auxiliary variables β and γ and rewrite Eq. (1) as:

$$\begin{aligned} \min_{\alpha,\beta,\gamma} \quad & \|\gamma\|_1 + \theta \|\beta\|_1 \\ \text{s.t.} \quad & \gamma = x - M(B\alpha + \mu), \quad \alpha = \beta, \\ & \|C_i (B\alpha + \mu)\|_2^2 = L_i, i = 1, \cdots, m. \end{aligned} \tag{2}$$

The augmented Lagrangian function of Eq. (2) is:

$$\begin{aligned} & \mathcal{L}_1(\alpha, \beta, \gamma, \lambda_1, \lambda_2, \eta) = \|\gamma\|_1 + \theta \|\beta\|_1 + \\ & \lambda_1^T [\gamma - x + M(B\alpha + \mu)] + \lambda_2^T (\alpha - \beta) + \\ & \tfrac{\eta}{2} \left[\|\gamma - x + M(B\alpha + \mu)\|^2 + \|\alpha - \beta\|^2 \right] \end{aligned}$$

where λ_1 and λ_2 are the Lagrange multipliers and $\eta > 0$ is the penalty parameter. ADM is to update the variables by minimizing the augmented Lagrangian function w.r.t. the variables alternately. In the following, k and l are the indices of iterations.

1.1. Update γ

We discard the terms in $\mathcal{L}_1$ which are independent of γ and update γ by:

$$\gamma^{k+1} = \underset{\gamma}{\operatorname{argmin}} \|\gamma\|_1 + \frac{\eta_k}{2} \left\| \gamma - \left[x - M(B\alpha^k + \mu) - \frac{\lambda_1^k}{\eta_k} \right] \right\|^2$$

which has a closed form solution [1].

1.2. Update β

We drop the terms in $\mathcal{L}_1$ which are independent of β and update β by:

$$\beta^{k+1} = \underset{\beta}{\operatorname{argmin}} \|\beta\|_1 + \frac{\eta_k}{2\theta} \left\| \beta - \left(\frac{\lambda_2^k}{\eta_k} + \alpha^k \right) \right\|^2$$

which also has a closed form solution [1].

1.3. Update α

We dismiss the terms in $\mathcal{L}_1$ which are independent of α and update α by:

$$\begin{aligned} \alpha^{k+1} = \arg\min_{\alpha} \quad & z^T W z \\ \text{s.t.} \quad & z^T \Omega_i z = 0, \quad i = 1, \cdots, m \end{aligned} \tag{3}$$

where $z = [\alpha^T \quad 1]^T$,

$$W = \begin{pmatrix} B^T M^T M B + I & 0 \\ 2\left[\left(\gamma^{k+1} - x + M\mu + \frac{\lambda_1^k}{\eta_k} \right)^T MB - \beta^{k+1} + \frac{\lambda_2^k}{\eta_k} \right] & 0 \end{pmatrix}$$

and $\Omega_i = \begin{pmatrix} B^T C_i^T C_i B & B^T C_i^T \mu \\ \mu^T C_i^T C_i B & \mu^T C_i^T C_i \mu - L_i \end{pmatrix}$.

Let $Q = zz^T$. Then the objective function becomes $z^T W z = \text{tr}(WQ)$ and Eq. (3) is transformed to:

$$\begin{aligned} \min_{Q} \quad & \text{tr}(WQ) \\ \text{s.t.} \quad & \text{tr}(\Omega_i Q) = 0, \quad i = 1, \cdots, m, \\ & Q \succeq 0, \quad \text{rank}(Q) \leq 1. \end{aligned} \tag{4}$$

We still solve problem (4) by the alternating direction method [1]. We introduce an auxiliary variable P and rewrite the problem as:

$$\begin{aligned} \min_{Q,P} \quad & \text{tr}(WQ) \\ \text{s.t.} \quad & \text{tr}(\Omega_i Q) = 0, \quad i = 1, \cdots, m, \\ & P = Q, \quad \text{rank}(P) \leq 1, \quad P \succeq 0. \end{aligned} \tag{5}$$

Its augmented Lagrangian function is:

$$\mathcal{L}_2(Q, P, G, \delta) = \text{tr}(WQ) + \text{tr}(G^T(Q - P)) + \frac{\delta}{2} \|Q - P\|_F^2$$

where G is the Lagrange Multiplier and $\delta > 0$ is the penalty parameter. We update Q and P alternately.

- Update Q:

$$Q^{l+1} = \underset{\substack{\mathrm{tr}(\Omega_i Q) = 0, \\ i = 1, \cdots, m}}{\mathrm{argmin}} \mathcal{L}_2(Q, P^l, G^l, \delta_l). \quad (6)$$

  This is a constrained least square problem and has a closed form solution.

- Update P: We discard the terms in $\mathcal{L}_2$ which are independent of P and update P by:

$$P^{l+1} = \underset{\substack{P \succeq 0, \\ \mathrm{rank}(P) \leq 1}}{\mathrm{argmin}} \left\| P - \tilde{Q} \right\|_F^2 \quad (7)$$

  where $\tilde{Q} = Q^{l+1} + \frac{2}{\delta_l} G^l$. Note that $\left\| P - \tilde{Q} \right\|_F^2$ is equal to $\left\| P - \frac{\tilde{Q}^T + \tilde{Q}}{2} \right\|_F^2$. Then (7) has a closed form solution by the lemma 1.1.

- Update G: We update the Lagrangian multiplier G by:

$$G^{l+1} = G^l + \delta^l (Q^{l+1} - P^{l+1}) \quad (8)$$

- Update δ: We update the penaly parameter by:

$$\delta^{l+1} = \min(\delta^l \cdot \rho, \delta^{max}), \quad (9)$$

  where $\rho \geq 1$ and δ^{max} are constant parameters.

Lemma 1.1 *The solution to*

$$\min_P \|P - S\|_F^2 \quad s.t. \quad P \succeq 0, \quad rank(P) \leq 1 \quad (10)$$

is $P = \max(\xi_1, 0)\nu_1\nu_1^T$, where S is a symmetric matrix and ξ_1 and ν_1 are the largest eigenvalue and eigenvector of S, respectively.

Proof Since P is a symmetric semi-positive definite matrix and its rank is one, we can write P as: $P = \xi \nu \nu^T$, where $\xi \geq 0$. Let the largest eigenvalue of S be ξ_1, then we have $\nu^T S \nu \leq \xi_1, \forall \nu$. Then we have:

$$\begin{aligned} \|P - S\|_F^2 &= \|P\|_F^2 + \|S\|_F^2 - 2\mathrm{tr}(P^T S) \\ &\geq \xi^2 + \textstyle\sum_{i=1}^n \xi_i^2 - 2\xi\xi_1 \\ &= (\xi - \xi_1)^2 + \textstyle\sum_{i=2}^n \xi_i^2 \\ &\geq \textstyle\sum_{i=2}^n \xi_i^2 + \min(\xi_1, 0)^2 \end{aligned} \quad (11)$$

The minimum value can be achieved when $\xi = \max(\xi_1, 0)$ and $\nu = \nu_1$.

1.4. Update λ_1

We update the Lagrangian multiplier λ_1 by:

$$\lambda_1^{k+1} = \lambda_1^k + \eta^k \left(\gamma^{k+1} - x + M \left(B\alpha^{k+1} + \mu \right) \right) \quad (12)$$

1.5. Update λ_2

We update the Lagrangian multiplier λ_2 by:

$$\lambda_2^{k+1} = \lambda_2^k + \eta^k \left(\alpha^{k+1} - \beta^{k+1} \right) \quad (13)$$

1.6. Update η

We update the penalty parameter η by:

$$\eta^{k+1} = \min(\eta^k \cdot \rho, \eta^{max}), \quad (14)$$

where $\rho \geq 1$ and η^{max} are the constant parameters.

2. Camera Parameter Estimation

Given estimated 2D pose X and 3D pose Y, we estimate camera parameters by solving the following optimization problem:

$$\min_{m_1, m_2} \left\| X - \begin{pmatrix} m_1^T \\ m_2^T \end{pmatrix} Y \right\|_1, \quad \text{s.t.} \quad m_1^T m_2 = 0. \quad (15)$$

We introduce an auxiliary variable R and rewrite Eq. (15) as:

$$\begin{aligned} &\min_{R, m_1, m_2} \quad \|R\|_1 \\ &\text{s.t.} \quad R = X - \begin{pmatrix} m_1^T \\ m_2^T \end{pmatrix} Y, \quad m_1^T m_2 = 0. \end{aligned} \quad (16)$$

We still use ADM to solve problem (16). Its augmented Lagrangian function is:

$$\begin{aligned} &\mathcal{L}_3(R, m_1, m_2, H, \zeta, \tau) \\ = \; &\|R\|_1 + \mathrm{tr}\left(H^T \left[\begin{pmatrix} m_1^T \\ m_2^T \end{pmatrix} Y + R - X \right] \right) + \zeta(m_1^T m_2) \\ &+ \tfrac{\tau}{2} \left[\left\| \begin{pmatrix} m_1^T \\ m_2^T \end{pmatrix} Y + R - X \right\|_F^2 + \left(m_1^T m_2 \right)^2 \right] \end{aligned}$$

where H and ζ are Lagrange multipliers and $\tau > 0$ is the penalty parameter.

2.1. Update R

We discard the terms in $\mathcal{L}_3$ which are independent of R and update R by:

$$R^{k+1} = \underset{R}{\mathrm{argmin}} \|R\|_1 + \frac{\tau_k}{2} \left\| R + \begin{pmatrix} \left(m_1^k\right)^T \\ \left(m_2^k\right)^T \end{pmatrix} Y - X + \frac{H^k}{\tau_k} \right\|_F^2$$

which has a closed form solution [1].

2.2. Update m_1

We discard the terms in $\mathcal{L}_3$ which are independent of m_1 and update m_1 by:

$$m_1^{k+1} = \underset{m_1}{\text{argmin}} \left\| \begin{pmatrix} m_1^T \\ \left(m_2^k\right)^T \end{pmatrix} Y + R^{k+1} - X + \frac{H^k}{\tau_k} \right\|_F^2 + \left(m_1^T m_2^k + \frac{\zeta^k}{\tau_k} \right)^2$$

This is a least square problem and has a closed form solution.

2.3. Update m_2

We discard the terms in $\mathcal{L}_3$ which are independent of m_2 and update m_2 by:

$$m_2^{k+1} = \underset{m_2}{\text{argmin}} \left\| \begin{pmatrix} \left(m_1^{k+1}\right)^T \\ m_2^T \end{pmatrix} Y + R^{k+1} - X + \frac{H^k}{\tau_k} \right\|_F^2 + \left(\left(m_1^{k+1}\right)^T m_2 + \frac{\zeta^k}{\tau_k} \right)^2$$

This is a least square problem and has a closed form solution.

2.4. Update H

We update Lagrange multiplier H by:

$$H^{k+1} = H^k + \tau^k \left(\begin{pmatrix} \left(m_1^{k+1}\right)^T \\ \left(m_2^{k+1}\right)^T \end{pmatrix} Y + R^{k+1} - X \right) \quad (17)$$

2.5. Update ζ

We update the Lagrange multiplier ζ by:

$$\zeta^{k+1} = \zeta^k + \tau^k \cdot \left(m_1^{k+1}\right)^T m_2^{k+1} \quad (18)$$

2.6. Update penalty parameter τ

We update the penaly parameter τ by:

$$\tau^{k+1} = \min(\tau^k \cdot \rho, \tau^{max},) \quad (19)$$

where $\rho \geq 1$ and τ^{max} are constant parameters.

References

[1] R. Liu, Z. Lin, and Z. Su. Linearized alternating direction method with parallel splitting and adaptive penalty for separable convex programs in machine learning, 2013.