Supplementary Material of PDO-eConvs: Partial Differential Operator Based Equivariant Convolutions

1. Numerical Schemes of Partial Differential Operators

1.1. Filters of Size 3×3

$$\widetilde{u}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_x = \frac{1}{h}\begin{bmatrix} 0 & 0 & 0 \\ -1/2 & 0 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_y = \frac{1}{h}\begin{bmatrix} 0 & 1/2 & 0 \\ 0 & 0 & 0 \\ 0 & -1/2 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xx} = \frac{1}{h^2}\begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xy} = \frac{1}{h^2}\begin{bmatrix} -1/4 & 0 & 1/4 \\ 0 & 0 & 0 \\ 1/4 & 0 & -1/4 \end{bmatrix}$$

$$\widetilde{u}_{yy} = \frac{1}{h^2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxy} = \frac{1}{h^3}\begin{bmatrix} 1/2 & -1 & 1/2 \\ 0 & 0 & 0 \\ -1/2 & 1 & -1/2 \end{bmatrix}$$

$$\widetilde{u}_{xyy} = \frac{1}{h^3}\begin{bmatrix} -1/2 & 0 & 1/2 \\ 1 & 0 & -1 \\ -1/2 & 0 & 1/2 \end{bmatrix}$$

$$\widetilde{u}_{xxyy} = \frac{1}{h^4}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

1.2. Filters of Size 5×5

$$\widetilde{u}_{xxx} = \frac{1}{h^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & -1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{yyy} = \frac{1}{h^3}\begin{bmatrix} 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1/2 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxxx} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xxxy} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1/4 & 1/2 & 0 & -1/2 & 1/4 \\ 0 & 0 & 0 & 0 & 0 \\ 1/4 & -1/2 & 0 & 1/2 & -1/4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{u}_{xyyy} = \frac{1}{h^4}\begin{bmatrix} 0 & -1/4 & 0 & 1/4 & 0 \\ 0 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 1/4 & 0 & -1/4 & 0 \end{bmatrix}$$

$$\widetilde{u}_{yyyy} = \frac{1}{h^4}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

2. The Topology of CNN Model

The topology of CNN model used in Section 5.1 is shown in Table 1. We replace each conventional convolution by a PDO-eConv and obtain an equivariant CNN model. The numbers of filters are modified accordingly, in order to keep the numbers of parameters nearly the same. Each PDO-eConv is followed by a batch normalization layer and an ReLU function. Particularly, batch normalization should be implemented with a single scale and a single bias per PDO-eConv map to preserve equivariance. We use dropout after PDO-eConvs and set the dropout rate to 0.2.

Table 1. The topology of the conventional CNN used in Section 5.1.

Layer	Parameters and channel size.
input	size: 28×28
convolution	channel: 20
convolution	channel: 20
max pooling	kernel: 2×2, stride: 2
convolution	channel: 20
convolution	channel: 20
convolution	channel: 20
convolution	channel: 20
linear	channel: 10
softmax	

3. PDO-eConv vs. Competitive Models

We compare the performance of our PDO-eConv with some more competitive models, using a larger model. The architecture is given in Table 2. Different from the architecture shown in Table 1, we use the orientation pooling after the final PDO-eConv layer, in order to get rotation-invariant features. Following (Weiler et al., 2018), we augment the dataset with continuous rotations during training time. For using data augmentation and the larger model, we train our model for 300 epochs, starting with a learning rate of 10^{-3} and reducing it gradually to 10^{-6}. The other training details are the same as that in Section 5.1. As shown in Table 3, SFCNN achieved 0.714% test error on rotated MNIST. Compared with SFCNN, our method achieves a comparable result, 0.709% test error, using only 10% parameters. To be specific, our method uses 0.65M parameters, while SFCNN needs 6.5M parameters.

Table 2. The topology of the CNN model using PDO-eConvs.

Layer	Parameters and channel size.
input	size: 28×28
PDO-eConv	channel: 16
PDO-eConv	channel: 16
max pooling	kernel: 2×2, stride: 2
PDO-eConv	channel: 32
PDO-eConv	channel: 32
PDO-eConv	channel: 32
PDO-eConv	channel: 64
PDO-eConv	channel: 64
max pooling	kernel: 2×2, stride: 2
orientation pooling	
linear	channel: 10
softmax	

Table 3. Error rates on MNIST-rot-12k (median of 5 runs).

Method	Test Error (%)
H-Net (Worrall et al., 2017)	1.69
OR-TIPooling (Zhou et al., 2017)	1.54
RotEqNet (Marcos et al., 2017)	1.09
RotEqNet (test time augmentation) (Marcos et al., 2017)	1.01
SFCNN (HeInit) (Weiler et al., 2018)	0.957
SFCNN (CoeffInit) (Weiler et al., 2018)	0.714
PDO-eConv (ours)	**0.709**

References

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