Optimizing Marketing Subsidies via Counterfactual Learning with Asymmetric Reward Function

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Abstract

In marketing, optimizing personalized subsidy allocation to maximize overall profits is of substantial economic importance. Prior research has employed treatment effect estimation techniques to identify subsidy-sensitive groups and design corresponding allocation strategies. However, more accurate treatment effect estimations do not necessarily lead to better allocations, underscoring the critical influence of decision boundaries in decision-making. This paper argues that optimal allocation fundamentally depends on predicting the expected minimum subsidy, a challenge distinct from conventional treatment effect estimation or causal decision-making, which existing approaches fail to address. To fill this gap, we introduce a two-stage **Co**unterfactual optimal subsidy **L**earning method with an **A**symmetric reward (**CoLA**). In the first stage, we derive a coarse estimate of the expected subsidy threshold by exploiting order information and the conditional independence between expected and observed subsidies. In the second stage, we refine these estimates using an asymmetric loss function, leading to more robust predictions. Under practical budget constraints, we prioritize candidates based on their Sharpe ratios to determine the final subsidy allocation strategy. Experiments on three public datasets and an online A/B test show that our method achieves significant performance improvements, yielding the highest total profit and incremental leverage ratios.

CCS Concepts

• **Information systems** → **Information retrieval**; • **Computing methodologies** → **Machine learning**; • **Applied computing** → **Electronic commerce**.

Keywords

Online Subsidy Allocation, Uplift Model, Causal Effect Estimation

ACM Reference Format:
Xiang Li, Yanghao Xiao, Chunyuan Zheng, Qian Zou, Bing Cheng, Wei Lin, Haoxuan Li*, and Zhouchen Lin*. 2026. Optimizing Marketing Subsidies via Counterfactual Learning with Asymmetric Reward Function. In *Proceedings of the 49th International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR '26), July 20–24, 2026, Melbourne, VIC, Australia.* ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3805712.3809750

1 Introduction

Advanced marketing technologies are increasingly adopted across industries, with online platforms using subsidies such as coupons or cash incentives to actively stimulate consumer transactions [10, 35, 61, 64]. A key challenge in marketing is determining how to allocate the budget for personalized subsidies to achieve the highest returns [73, 74]. In essence, subsidy allocation is closely coupled with recommendation systems [29, 30, 39, 60, 69], as the recommender controls item exposure while the subsidy module determines the attached incentive, and both jointly drive user conversion. An effective subsidy allocation aims to predict the optimal subsidy for each individual so as to maximize the platform's total profit. To address the problem, prior research building on causal inference principles conceptualizes subsidies as treatment and predicts treatment effects to identify groups that benefit most [19, 36, 70]. Based on the identified marketing-sensitive groups, it enables the development of personalized marketing strategies that enhance the efficiency of subsidy allocation for individuals to achieve maximum revenue.

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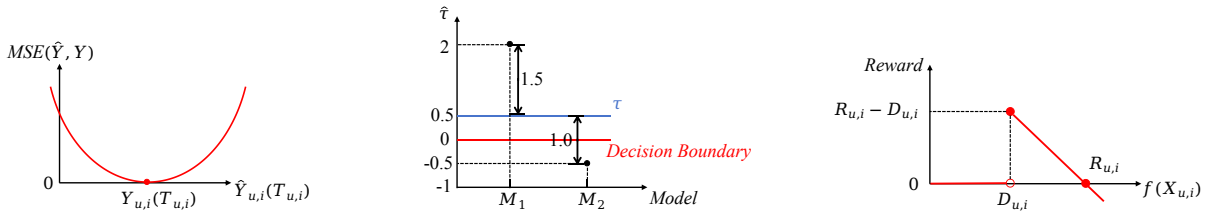
SIGIR '26, Melbourne, VIC, Australia.

ACM ISBN 979-8-4007-2599-9/2026/07
https://doi.org/10.1145/3805712.3809750



(a) Treatment Effect Estimation. (b) Causal Decision Making. (c) Optimal Subsidy Prediction.

Figure 1: The comparison of treatment effect estimation, causal decision making and optimal subsidy prediction problem. (a) In treatment effect estimation, symmetric regression loss is used for potential outcome modeling. (b) In causal decision making, decision boundary plays a more crucial role, where τ and $\hat{\tau}$ are the true and estimated causal effect, respectively. (c) In optimal subsidy prediction, the target is to estimate expected subsidy $D_{u,i}$ for profit maximization.

Prior studies simplify subsidy allocation to treatment effect estimation, focusing on accurate effect prediction. For instance, in binary settings where treatment is defined as whether to allocate a subsidy, various causal inference methods have been applied, including propensity score based methods [9, 44, 46], tree based methods [7, 17, 21], representation learning methods [24, 48], and generative model based methods [37, 62]. In continuous treatment scenarios, where the treatment is defined as the specific amount of subsidy allocated, also known as dose-response curve estimation, existing methods include Generalized Propensity Score (GPS) based methods [38, 42], DRNet [47], VCNet [40], and SCIGAN [5]. Based on predicted treatment effects, users are grouped and subsidies are allocated only to those with positive profit gains.

However, recent literature suggests that more accurate treatment effect estimations do not necessarily improve the decision making [12]. The key insight is that causal decision making belongs to a classification task based on decision boundaries, while treatment effect estimation is a regression task towards the ground truth effect [13]. In this work, we argue that optimal subsidy prediction in marketing is fundamentally neither a treatment effect estimation problem nor a causal decision-making problem. The essence of this task is to predict the expected subsidy threshold, specifically the minimum subsidy required to incentivize user behavior. Concretely, if the predicted subsidy falls below a certain threshold, the user will not place an order, resulting in zero reward, while if it exceeds the expected subsidy threshold, the user still makes a purchase, but the reward diminishes as the subsidy increases. In summary, optimal subsidy prediction relies on an asymmetric reward function based on the expected minimum subsidy, whereas treatment effect estimation is grounded in symmetric regression losses, and causal decision making centers on identifying decision boundaries. We illustrate the distinctions among these three problems in Figure 1. Recognizing these differences, existing methods based on treatment effect estimation or causal decision-making cannot achieve optimal subsidy prediction.

To address this gap, we propose a two-stage Counterfactual robust optimal subsidy **L**earning framework with an **A**symmetric reward function (**CoLA**). Note that precisely estimating the expected subsidy threshold is inherently challenging, our method focuses on generating robust subsidy predictions that ideally fall within the right-side neighborhood of this unknown threshold. In the first stage, we integrate order information with the conditional independence between expected and observed subsidies to obtain a coarse estimate of the expected subsidy threshold. In the second stage, we refine these estimates using an asymmetric loss function, intentionally producing slightly overestimated predictions. This design encourages predictions to reside in the right-side neighborhood of the true minimum expected subsidy. Given practical budget constraints, we compute the Sharpe ratio of predicted subsidies to measure unit return and allocate subsidies to candidates with the highest ratios. We conduct semi-simulated experiments on three public datasets and an online A/B test. Results demonstrate that the proposed method consistently outperforms existing causal effect estimation approaches, delivering superior subsidy allocation strategies. The key contributions of this study are summarized as follows.

- To the best of our knowledge, this is the first work to highlight that optimal subsidy allocation in marketing fundamentally relies on predicting the expected subsidy threshold and cannot be reduced to a treatment effect estimation or causal decision-making problem.
- We propose a two-stage counterfactual robust optimal subsidy learning method grounded in asymmetric reward function to produce robust predictions expected to lie within the right-side neighborhood of the true expected threshold.
- Extensive experiments on three public datasets and an online A/B test demonstrate that our method surpasses prior treatment effect estimation-based approaches, delivering superior subsidy allocation strategies.

2 Related Work

Causal Effect Estimation and Decision-Making. For estimating treatment effect [58, 68, 71, 72], a wide body of literature has focused on binary treatment scenarios, which can be broadly categorized into tree-based methods [7, 17, 21, 53], propensity score-based methods [9, 44–46, 51], covariate balancing methods [3, 11, 23, 57], doubly robust methods [4, 41, 43], representation learning methods [8, 24, 25, 36, 48, 50, 70], and generative model based methods [37, 62]. When the treatment is continuous variable such as subsidy value, estimating the treatment effect is commonly referred to as dose-response curve estimation. By modeling conditional density

of treatment given features, the Generalized Propensity Score (GPS)-based methods adopt matching [38] and weighting [42] for treatment effect estimation. DRNet extends traditional representation-based methods to continuous treatment settings, which discretizes the continuous treatment values into multiple intervals and models each interval separately [47]. VCNet takes one step further by introducing a varying coefficient prediction head network [40], guaranteeing the continuity in the predicted dose-response curve. SCIGAN extends the traditional generative modeling methods, which introduces both treatment type discriminator and dosage discriminators to improve the quality of the generated counterfactuals through adversarial learning [5]. Different from causal effect estimation, causal decision making aims to determine whether a specific treatment should be applied to a unit to achieve an optimal outcome. For binary treatments, the goal is to identify individuals whose outcomes would change from negative to positive if treated [12]. Fernández-Loría and Provost [13] emphasize that accurate effect estimation is not always necessary for effective causal decision making. The key difference is that causal decision making is a classification problem, while causal effect estimation is a regression task [63, 66].

Subsidy Allocation in Marketing. In recent years, subsidy allocation methods have gained widespread attention in marketing, which share close connections with recommender systems [28, 31, 34, 55], as both aim to estimate heterogeneous user responses and optimize personalized treatment [32, 54, 56, 59]. Traditional heuristic approaches [16, 65, 67] focus on predicting uplift effects but lack a clear optimization framework, limiting their effectiveness. The mainstream budget allocation method is a two-stage approach [1, 15], where causal inference is used in the first stage to predict treatment effects, and integer programming is used in the second stage for optimal allocation. Based on this, Li et al. [33] propose a personalized optimization method based on counterfactual estimation and multi-task learning, while Chen et al. [6] estimate treatment effects unbiasedly using inverse propensity weighting and optimize exposure strategies. He et al. [19] address the long-tail distribution problem in income regression by normalizing and adjusting extreme data points. Sun et al. [52] improve model robustness by solving the issue of over-reliance on features through adversarial desensitization, and Huang et al. [22] address treatment misalignment by integrating contextual information. Note that existing marketing algorithms typically use symmetric loss functions like MSE for treatment effect estimation. However, more accurate causal effect estimates do not guarantee a better subsidy strategy. To address this issue, we develop a two-stage subsidy learning method to achieve better subsidy allocation.

3 Problem Setup

Define $\mathcal{U} = \{u_1, \ldots, u_m\}$ be the user set, $\mathcal{I} = \{i_1, \ldots, i_n\}$ be the item set, and $\mathcal{O} = \mathcal{U} \times \mathcal{I} = \{(u_1, i_1), \ldots, (u_m, i_n)\}$ be the set of all user-item pairs. For each user-item pair (u, i), denote $X_{u,i} \in \mathbb{R}^P$ be the observed features, $T_{u,i} \in \mathbb{R}^+ \cup \{0\}$ be the subsidy amount, and $Y_{u,i} \in \mathbb{R}$ be the profit. We adopt the potential outcome framework, in which the subsidy $T_{u,i} \geq 0$ is defined as the treatment, and the potential outcome $Y_{u,i}(t)$ denotes the profit that would be observed if the subsidy $T_{u,i}$ were set to t. In practice, each user-item pair can only receive one treatment, resulting in only one observed outcome, which is known as the fundamental problem of causal inference. We assume the consistency assumption holds, meaning that the observed outcome corresponds to the potential outcome under the actual treatment, i.e., $Y_{u,i} = Y_{u,i}(T_{u,i})$. Additionally, we assume the Stable Unit Treatment Value Assumption (SUTVA) holds, which states that there are no alternative forms of treatment and that user-item pairs are independent. Denote $\mathcal{O}_{tr}$ and $\mathcal{O}_{te}$ be the training and test set.

In the marketing scenario, our goal is to incentivize user purchases through subsidies. Let $D_{u,i} \in \mathbb{R}^+ \cup \{0\}$ denote the expected minimum subsidy threshold where user u would purchase item i. When the actual subsidy $T_{u,i} \geq D_{u,i}$, the user makes a purchase, otherwise, the user does not purchase, and the potential outcome $Y_{u,i}(T_{u,i})$ can be expressed as follows:

$$Y_{u,i}(T_{u,i}) = \begin{cases} R_{u,i} - T_{u,i}, & \text{if } T_{u,i} \geq D_{u,i}, \\ 0, & \text{if } T_{u,i} < D_{u,i}, \end{cases} \tag{1}$$

where $R_{u,i} \in \mathbb{R}^+$ is the known net profit without subsidy.

Ideally, the optimal treatment is the expected minimum subsidy threshold required to trigger the user u to buy item i, i.e., $T^*_{u,i} = D_{u,i}$. An ideal goal is to learn a prediction model that accurately estimates the unknown $D_{u,i}$ from observed features $X_{u,i}$. However, given the difficulty of precise point estimation, this study instead aims to learn a prediction function $f : \mathbb{R}^P \to \mathbb{R}^+ \cup \{0\}$ such that the predicted value $f(X_{u,i})$ falls in the right-side neighborhood of the unknown $D_{u,i}$, achieving robust predictions by satisfying $f(X_{u,i}) \geq D_{u,i}$ while remaining close to $D_{u,i}$. Random variables are denoted without subscripts.

4 Methodology

4.1 Two-stage Learning Method

We decompose the robust subsidy prediction task into two stages. In Stage 1, we use historical subsidy and purchase information to obtain a coarse estimate of the expected subsidy threshold. In Stage 2, we refine this estimate with an asymmetric loss that encourages slight overestimation, guiding the final predictions fall within the right-side neighborhood of the unknown true expected subsidy threshold and thereby enhancing robustness.

4.1.1 Stage 1: Coarse-grained Expected Subsidy Estimation. In the first stage, we develop an auxiliary expected subsidy estimation model $f_D(X_{u,i}) = \hat{D}_{u,i}$, designed to estimate the expected subsidy threshold $D_{u,i}$, distinct from the final prediction model f. The supervision signal is derived from two sources.

- First, by leveraging historical subsidy and purchase data in the training set, we can infer a valid range for the expected subsidy, serving as a factual supervision signal.
- Second, since a user's minimum expected subsidy should be fully determined by features and conditionally independent of historical subsidies, we introduce a regularizer to enforce the conditional independence of D and T given X as an additional supervision signal.

Based on the collected training data, if a user orders $Y_{u,i} > 0$ when receiving a subsidy $T_{u,i}$, it indicates that the expected subsidy $D_{u,i}$ is less than the factual subsidy $T_{u,i}$. Conversely, if the user does

not order $Y_{u,i} = 0$, it suggests that the expected subsidy $D_{u,i}$ exceeds the factual subsidy $T_{u,i}$. According to this, we propose the following hinge loss:

$$L_{\text{hinge}}(f_D) = \frac{1}{|O_{tr}|}\Big[\sum_{Y_{u,i}=0} \max(T_{u,i} - f_D(X_{u,i}), 0) + \sum_{Y_{u,i}>0} \max(f_D(X_{u,i}) - T_{u,i}, 0)\Big]. \quad (2)$$

On the other hand, a user's psychological expected subsidy D typically depends only on user and item features (e.g., income level, item price) and is independent of the actual subsidy T assigned by either an algorithm or random allocation. Formally, this implies that the conditional covariance is zero, i.e., $\mathbb{E}[(D-\mathbb{E}[D \mid X]) \cdot (T-\mathbb{E}[T \mid X]) \mid X] = 0$, and we propose using this conditional covariance to measure the target conditional dependence.

In practice, $\mathbb{E}[D \mid X]$ is modeled by the target function f_D, while $\mathbb{E}[T \mid X]$ is implemented as an auxiliary function f_T, which takes features $X_{u,i}$ as input to predict the assigned subsidy (treatment) $T_{u,i}$. Since $D_{u,i}$ is unobserved, direct computation of the conditional covariance is infeasible. To address this, we generate pseudo-labels for $D_{u,i}$ by scaling the net profit without subsidy, $R_{u,i}$, by a factor k, reflecting the intuition that higher product margins (prices) imply higher expected subsidies. Based on this construction, the conditional dependence loss is derived as:

$$\begin{aligned} &L_{\text{cd}}(f_D, f_T) \\ &= \frac{1}{|O_{tr}|} \sum_{(u,i)\in O_{tr}} (\bar{D}_{u,i} - f_D(X_{u,i})) \cdot (T_{u,i} - f_T(X_{u,i})) \\ &= \frac{1}{|O_{tr}|} \sum_{(u,i)\in O_{tr}} (k \cdot R_{u,i} - f_D(X_{u,i})) \cdot (T_{u,i} - f_T(X_{u,i})), \end{aligned} \quad (3)$$

where $\bar{D}_{u,i} = k \cdot R_{u,i}$ is the pseudo-label, and $0 < k < 1$ is a hyper-parameter. The auxiliary model f_T is jointly optimized using conditional dependence loss in Equation 3.

In summary, in Stage 1, the final loss function $L_{\text{1-stage}}$ of the expected subsidy model f_D is:

$$L_{\text{1-stage}}(f_D, f_T) = L_{\text{hinge}}(f_D) + \alpha \cdot L_{\text{cd}}(f_D, f_T), \quad (4)$$

where α is a hyper-parameter. The learned model f_D provides a coarse-grained estimate of $D_{u,i}$ denoted as $\hat{D}_{u,i}$.

4.1.2 Stage 2: Robust Prediction Based on Asymmetric Loss. Since perfectly predicting the unobserved expected subsidy is highly challenging or even impossible, the first-stage output $\hat{D}_{u,i} = f_D(X_{u,i})$ inevitably contains errors. In other words, using $\hat{D}_{u,i}$ as the final subsidy prediction leads to zero reward when $\hat{D}_{u,i}$ falls slightly below the unknown true threshold $D_{u,i}$, whereas predictions slightly above it still yield acceptable returns. This motivates the use of an asymmetric loss function that deliberately biases predictions upward to ensure final prediction $f(X_{u,i}) \geq \hat{D}_{u,i}$, providing a more robust, "better safe than sorry" subsidy.

Given the estimated expected subsidy $\hat{D}_{u,i}$ learned from Stage 1, a natural idea is to use the naive negative reward function shown in Figure 1 (c) as the Stage-2 loss, given as:

$$-\hat{R}(f) = \frac{1}{|O_{tr}|} \sum_{(u,i)\in O_{tr}} (f(X_{u,i}) - R_{u,i}) \mathbb{I}(f(X_{u,i}) \geq \hat{D}_{u,i}). \quad (5)$$

However, directly optimizing the negative reward function $-\hat{R}(f)$ has two main drawbacks. First, the objective function is discontinuous at $f(X_{u,i}) = \hat{D}_{u,i}$. Second, when $f(X_{u,i}) < \hat{D}_{u,i}$, the objective function value always remains zero, thereby blocking gradient flow and preventing effective parameter updates.

To address these issues, we introduce the following approximation function to replace the indicator function $\mathbb{I}(\cdot)$:

$$g(x) = \begin{cases} x + \hat{D}_{u,i} - R_{u,i}, & \text{if } x \geq 0, \\ -|\hat{D}_{u,i} - R_{u,i}| \cdot e^{\tau \cdot x}, & \text{if } x < 0, \end{cases} \quad (6)$$

where $\tau > 0$ is a large constant. Based on this, we define the refinement loss $L_{\text{2-stage}}(f)$ as:

$$L_{\text{2-stage}}(f) = \frac{1}{|O_{tr}|} \sum_{(u,i)\in O_{tr}} g(f(X_{u,i}) - \hat{D}_{u,i}). \quad (7)$$

Using $L_{\text{2-stage}}(f)$ loss for training possesses several desired properties. First, this construction ensures continuity at $f(X_{u,i}) = \hat{D}_{u,i}$. Second, it generates nonzero gradients whenever $f(X_{u,i}) < \hat{D}_{u,i}$, guiding the model to satisfy $f(X_{u,i}) \geq \hat{D}_{u,i}$. Third, as $\tau \to \infty$, the $L_{\text{2-stage}}(f)$ loss converges to the original negative reward function, thus preserving the crucial asymmetric penalty property.

The resulting model f then produces the final robust subsidy predictions. In the absence of budget constraints, these predictions directly constitute the subsidy policy for each user-item pair.

4.2 Theoretical Analysis

We now establish theoretical guarantees for our two-stage learning framework. Our goal is to prove that the regret of the learned policy relative to the ideal optimal policy is bounded under finite samples.

Define the reward function for any prediction function f as:

$$\pi(f) = \mathbb{E}[(R_{u,i} - f(X_{u,i}))\mathbb{I}(f(X_{u,i}) \geq D_{u,i})], \quad (8)$$

where $\pi(f)$ represents the expected profit, $R_{u,i}$ is the net profit without subsidy, $f(X_{u,i})$ is the predicted subsidy, $D_{u,i}$ is the true expected subsidy threshold, and $\mathbb{I}(\cdot)$ is the indicator function.

The regret is defined as:

$$\text{Regret}(f(\hat{D})) = \pi(f^*) - \pi(f(\hat{D})), \quad (9)$$

where f^* is the ideal optimal policy with access to true $D_{u,i}$, and $f(\hat{D})$ is our learned policy based on estimated $\hat{D}_{u,i}$ from Stage 1.

Theorem 4.1 (Regret Bound). *With probability at least $1-\delta$, the regret satisfies:*

$$\pi(f^*) - \pi(f(\hat{D})) \leq 2(1 + \max_{u,i} |\hat{D}_{u,i} - R_{u,i}|) \cdot Ra(\mathcal{F}_D) + 4T_{\max}\sqrt{\frac{2\log(4/\delta)}{|\mathcal{U}||\mathcal{I}|}},$$

where the Rademacher complexity $Ra(\mathcal{F}_D)$ measures the function class capacity, defined as:

$$Ra(\mathcal{F}_D) = \mathbb{E}_{\sigma\sim\{-1,+1\}^{|\mathcal{U}||\mathcal{I}|}}\left[\sup_{f_D\in\mathcal{F}_D} \frac{1}{|\mathcal{D}|}\sum_{(u,i)} \sigma_{u,i}\mathcal{L}_{hinge}(f_D(X_{u,i}))\right],$$

where $\sigma_{u,i}$ are independent Rademacher random variables taking values in $\{-1,+1\}$ with equal probability, and $\mathcal{L}_{hinge}$ is the hinge loss from Stage 1. $\max_{u,i} |\hat{D}_{u,i} - R_{u,i}|$ captures the worst-case estimation bias, $T_{\max}$ is the maximum subsidy value, $|\mathcal{U}|$ and $|\mathcal{I}|$ are the numbers of users and items, and δ is the confidence parameter.

The regret bound consists of two terms. The first term $2(1 + \max_{u,i} |\hat{D}_{u,i} - R_{u,i}|) \cdot Ra(\mathcal{F}_D)$ captures the estimation error from Stage 1: smaller Rademacher complexity and more accurate threshold estimates lead to smaller regret. The second term $4T_{\max}\sqrt{\frac{2\log(4/\delta)}{|\mathcal{U}||\mathcal{I}|}}$ represents statistical fluctuation that decays at rate $O(1/\sqrt{n})$ as sample size increases.

This theorem establishes that our two-stage framework achieves provably near-optimal subsidy allocation with high probability. The bound demonstrates that: (1) accurate Stage 1 estimation (via hinge loss and conditional independence) directly improves final performance; (2) the asymmetric loss in Stage 2 provides robustness by ensuring predictions fall in the right-side neighborhood of true thresholds; and (3) the regret vanishes as sample size grows, guaranteeing consistency.

4.3 Allocation Strategy with Budget Constraint

In real-world subsidy allocation scenarios, decisions must always be made under a predefined budget constraint B. To this end, we propose a subsidy allocation strategy based on the Sharpe Ratio (SR) [49], which assumes each predicted subsidy $\hat{T}_{u,i}$ triggers a purchase and evaluates the profitability per unit of subsidy, given as:

$$SR(\hat{T}_{u,i}) = \frac{R_{u,i} - \hat{T}_{u,i}}{\hat{T}_{u,i}}, \tag{10}$$

where $R_{u,i} - \hat{T}_{u,i}$ is the expected profit under subsidy $\hat{T}_{u,i}$ and $\hat{T}_{u,i}$ is the subsidy cost. Based on this metric, the subsidy allocation strategy comprises two steps. First, compute the Sharpe ratio $\mathrm{SR}(\hat{T}_{u,i})$ for each predicted subsidy $\hat{T}_{u,i}$ and sort all user-item pairs in descending order. This ranking reflects the return on subsidy, where pairs at the top generate the greatest profit per unit cost. Second, allocate subsidies to pairs in this order until the cumulative subsidy expenditure reaches the budget constraint, and all remaining pairs receive a subsidy of zero. This procedure ensures that, under the budget constraint, total profit is maximized.

The proposed two-stage optimal subsidy prediction algorithm with budget constraints is shown in Algorithm 1. Lines 1-13 train the subsidy prediction model through two-stage learning, and Line 14 produces its output also known as the robust estimations of expected subsidy, yielding the subsidy policy without budget constraints. Lines 15-26 then apply the Sharpe Ratio procedure to derive the budget-constrained allocation strategy.

Algorithm 1: Two-stage Robust Optimal Subsidy Learning with Budget

Input: Subsidy prediction model f; Expected subsidy estimation model f_D; Factual subsidy prediction model f_T; Training dataset $\{X_{u,i}, T_{u,i}, Y_{u,i}, R_{u,i}\}_{(u,i)\in\mathcal{O}_{tr}}$; Test dataset$\{X_{u,i}, R_{u,i}\}_{(u,i)\in\mathcal{O}_{te}}$; Budget B.

1 $Epoch \leftarrow 0$;
2 **while** *not convergent* **do**
3 Compute hinge loss $L_{\text{hinge}}(f_D)$ and the conditional dependence loss $L_{\text{cd}}(f_D, f_T)$;
4 Compute the final loss of the first stage $L_{\text{1-stage}}(f_D, f_T)$;
5 Update f_D and f_T jointly using $L_{\text{1-stage}}(f_D, f_T)$;
6 $Epoch \leftarrow Epoch + 1$;
7 **end**
8 $Epoch \leftarrow 0$;
9 **while** *not convergent* **do**
10 Compute the subsidy prediction loss $L_{\text{2-stage}}(f)$;
11 Update f using $L_{\text{2-stage}}(f)$;
12 $Epoch \leftarrow Epoch + 1$;
13 **end**
14 Obtain optimal subsidy prediction $\{f(X_{u,i})\}_{(u,i)\in\mathcal{O}_{te}}$;
15 Compute Sharpe Ratio $\{SR(f(X_{u,i}))\}_{(u,i)\in\mathcal{O}_{te}}$;
16 Cost $C \leftarrow 0$, Subsidy prediction $\hat{T}_{u,i} \leftarrow 0$;
17 Sort the samples in descending order of their Sharpe Ratios;
18 **for** *each sample* $(u,i) \in \mathcal{O}_{te}$ *in sorted order* **do**
19 **if** $C + f(X_{u,i}) \le B$ **then**
20 $\hat{T}_{u,i} \leftarrow f(X_{u,i})$;
21 $C \leftarrow C + f(X_{u,i})$;
22 **end**
23 **else**
24 $\hat{T}_{u,i} \leftarrow 0$;
25 **end**
26 **end**

Output: $\{\hat{T}_{u,i}\}_{(u,i)\in\mathcal{O}_{te}}$.

5 Experiments

5.1 Experimental Setup

This study addresses a continuous treatment (subsidy) scenario, whereas existing public marketing datasets are configured for binary treatment and thus unsuitable. Accordingly, following prior work [33], we employ semi-synthetic data generated from public datasets, which is common in causal inference research. In the semi-synthetic experiments, the methods capable of handling continuous treatment are selected as baselines. Furthermore, we conduct an online A/B test to validate the effectiveness of the proposed method on a real-world demand-side platform with over 100 million daily active users.

5.1.1 Datasets and Semi-synthetic Data Generation. In this study, we conduct experiments on MovieLens-1M[1] [18], Yelp[2] [2] and KuaiRec[3] [14] datasets. Specifically, ML-1M contains 1,000,209 five-point scale ratings provided by 6,040 users for 3,952 items, Yelp comprises 731,671 five-point scale ratings provided by 25,677 users for 25,815 items, and KuaiRec comprises 4,676,570 video watching ratios recorded from 1,411 users across 3,327 videos. These datasets contain potential preference information that may reflect plausible economic relationships between user preferences and expected subsidies (e.g., higher preferences correspond to lower expected minimum subsidies).

For all datasets, we generate the following variables for each unit: potential gross profit $R_{u,i}$, real subsidy $T_{u,i}$, optimal subsidy $D_{u,i}$, and real profit $Y_{u,i}$ as follows.

[1] https://grouplens.org/datasets/movielens/1M/
[2] https://www.yelp.com/dataset
[3] https://github.com/chongminggao/KuaiRec

• **Generating potential gross profit** $R_{u,i}$: We pre-trained a Matrix Factorization (MF) [26] model to generate predicted ratings, and we treat the predicted values as the potential gross profit $R_{u,i}$.

• **Generating factual subsidy** $T_{u,i}$: The real subsidy $T_{u,i}$ follows a uniform distribution in the range $[0, R_{u,i}]$ with probability γ, and with probability $1-\gamma$, it takes the value 0. In this experiment, we set $\gamma = 0.95$.

• **Generating optimal subsidy** $D_{u,i}$: We pre-trained a neural collaborative filtering (NCF) [20] model to generate predicted ratings, denoted by $\hat{y}_{u,i}$. For the top 5% of samples based on $\hat{y}_{u,i}$, $D_{u,i}$ is set to 0. For the remaining samples, $D_{u,i}$ follows a Gamma distribution with shape parameter θ and scale parameter $K/\hat{y}_{u,i}$. For the MovieLens-1M, Yelp, and KuaiRec datasets, we set K to 4.5, 3.5, and 1, respectively, and for all datasets, we set $\theta = 2$.

• **Generating real profit** $Y_{u,i}$: Based on $T_{u,i}$ and $D_{u,i}$, the conversion is modeled as a non-deterministic (probabilistic) event to simulate a sparse, real-world scenario with low conversion rates. First, we define the conversion rate (CVR) using a scaled sigmoid function:

$$\mathrm{cvr}_{u,i} = \frac{1}{1000 \cdot (1 + \exp(-10 \cdot (T_{u,i} - D_{u,i})))}$$

This function models a permille-level probability of conversion that is dependent on the relationship between the allocated subsidy $T_{u,i}$ and the user's expected subsidy $D_{u,i}$. The final real profit $Y_{u,i}$ is then generated from a Bernoulli distribution:

$$Y_{u,i} = \begin{cases} R_{u,i} - T_{u,i}, & \text{with probability } \mathrm{cvr}_{u,i}, \\ 0, & \text{with probability } 1 - \mathrm{cvr}_{u,i}. \end{cases}$$

During the training phase, only $R_{u,i}$, $T_{u,i}$, and $Y_{u,i}$ are available, while $D_{u,i}$ is unavailable.

5.1.2 Baselines. We compare the proposed method with both heuristic subsidy allocation strategies and continuous treatment effect estimation methods, as outlined below:

• **Heuristic-None**: This strategy does not allocate any subsidies, i.e.,$T_{u,i} = 0, \forall (u,i) \in \mathcal{O}_{te}$. The resulting profit $Reward_0$ is derived solely from the group where $D_{u,i} = 0$.

• **Heuristic-Random**: For any user-item pair (u,i), this strategy randomly samples the subsidy $T_{u,i}$ from a uniform distribution over $[0, R_{u,i}]$.

• **S-Learner** [27]: This is a type of meta-learners that treat the observed treatment $T_{u,i}$ as a one-dimensional feature and it is concatenated with the existing features and used to estimate the outcome.

• **GPS (Generalized Propensity Score)** [42]: This method employs the generalized propensity score for continuous treatments to re-weight the observed samples.

• **DRNet** [47]: This method learns a shared representation, stratifies the continuous treatment $T_{u,i}$ into discrete intervals and trains separate prediction head models for each interval to estimate the outcome.

• **CRNet** [75]: This method learns both balancing and prognostic representations through a contrastive learning framework to achieve unbiased estimation of heterogeneous dose-response curves.

• **VCNet** [40]: Similar to DRNet, this method initially learns a shared representation, and then trains a prediction head model with varying coefficients, which incorporates the continuous treatment $T_{u,i}$ as an input, to estimate the outcome.

Treatment effect estimation methods, such as VCNet, are capable of predicting the outcome corresponding to specific features $X_{u,i}$ and a given treatment $T_{u,i}$. To address the subsidy prediction task, we extend their application by providing multiple candidate treatments and selecting the treatment that yields the highest predicted outcome as the optimal treatment. Specifically, for each test sample, the candidate interventions are defined as the K-equidistant points within the interval $[0, R_{u,i}]$, where K is set to 10 across all datasets. We will release all code for the semi-synthetic experiments upon the paper's acceptance.

5.1.3 Experimental Environment. The semi-synthetic experiments are carried out on an Ubuntu 22.04 system equipped with 80GB of memory, 32-core CPU, and 24GB NVIDIA 3090 GPUs, utilizing PyTorch 2.0.0 and CUDA 11.7. And the online A/B test experiments are implemented in TensorFlow and executed on NVIDIA A100 GPUs.

5.1.4 Evaluation Metrics. We adopt the following three evaluation metrics to assess the performance of the predicted subsidy strategy $\{T^{\text{pred}}_{u,i}\}_{(u,i)\in\mathcal{O}_{te}}$:

• **Order Rate (OR)** evaluates the proportion of successful purchases under the predicted subsidy strategy. In this study, the OR values are scaled by 10^{-3} (permille) to reflect practical industry standards, defined as:

$$OR = \frac{\sum_{(u,i)\in\mathcal{O}_{te}} \mathbb{I}(T^{\text{pred}}_{u,i} \geq D_{u,i}) \cdot \mathbb{I}(T^{\text{pred}}_{u,i} \leq R_{u,i})}{|\mathcal{O}_{te}|},$$

where $\mathbb{I}(\cdot)$ is the indicator function, $|\mathcal{O}_{te}|$ represents the total number of test samples, and $D_{u,i}$ and $R_{u,i}$ denote the optimal subsidy and the potential net profit, respectively.

• **Reward** measures the total profit achieved under the predicted subsidy strategy, which is calculated as:

$$Reward = \sum_{(u,i)\in\mathcal{O}_{te}} Y^{\text{pred}}_{u,i},$$

where $Y^{\text{pred}}_{u,i}$ is profit under subsidy $T^{\text{pred}}_{u,i}$, determined by:

$$Y^{\text{pred}}_{u,i} = \begin{cases} R_{u,i} - T^{\text{pred}}_{u,i}, & \text{if } T^{\text{pred}}_{u,i} \geq D_{u,i}, \\ 0, & \text{if } T^{\text{pred}}_{u,i} < D_{u,i}. \end{cases}$$

• **Incremental Leverage Ratio (ILR)** quantifies the incremental profit per unit subsidy relative to the no-subsidy strategy None method ($Reward_0$) and is expressed as:

$$ILR = \frac{Reward - Reward_0}{\sum_{(u,i)\in\mathcal{O}_{te}} T^{\text{pred}}_{u,i}}.$$

5.1.5 Implementation Details. We evaluate the performance of different methods under two scenarios: **with** and **without** budget constraints. In the budget-constrained (with budget) scenario, all models prioritize subsidy allocation to samples with the highest Sharpe Ratio, continuing this process until the cumulative subsidy expenditure reaches the predefined budget B. We define the budget B as follows:

$$B = \beta \cdot \sum_{(u,i)\in\mathcal{O}_{te}} R_{u,i},$$

Table 1: Performance comparison with and without budget, where the best results and baseline results are bolded and underlined. Asterisk (*) indicates a significant improvement over the best baseline VCNet using a paired t-test across 10 runs at $p < 0.05$. To align with real-world scenarios, the *OR* metric is reported in permille (10^{-3}).

With Budget	ML-1M			Yelp			KuaiRec		
Methods	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR
None	2.891	3212.553	-	2.956	2425.546	-	1.932	1099.504	-
Random	4.641	3754.941	0.511	5.801	2837.835	0.316	2.619	1108.885	0.324
S-Learner	4.371	4426.183	0.468	4.086	3357.425	0.298	1.934	1181.923	0.239
GPS	4.542	4610.557	0.469	6.150	3775.710	0.391	2.313	1149.406	0.481
DRNet	4.179	4905.412	0.354	7.319	4022.938	0.866	2.681	1180.909	0.855
CRNet	4.638	5120.958	1.244	6.962	4116.185	0.924	2.352	1283.901	1.046
VCNet	4.876	5450.379	1.348	6.972	3991.982	1.134	2.745	1423.771	1.281
	± 0.025	± 18.609	± 0.002	± 0.015	± 14.317	± 0.007	± 0.016	± 8.709	± 0.010
CoLA	**5.334***	**5573.352***	**1.588***	**8.728***	**4226.012***	**1.454***	**2.969***	**1471.073***	**1.533***
	± 0.052	± 20.197	± 0.014	± 0.016	± 15.086	± 0.005	± 0.024	± 6.636	± 0.020
Without Budget	ML-1M			Yelp			KuaiRec		
Methods	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR
None	2.891	3212.553	-	2.956	2425.546	-	1.932	1099.504	-
Random	4.819	4113.743	0.403	5.929	2603.875	0.308	2.660	1047.884	0.315
S-Learner	4.414	3840.658	0.468	4.484	3128.653	0.271	2.081	1159.666	0.230
GPS	4.874	4303.313	0.631	6.308	3502.993	0.386	2.323	1130.326	0.262
DRNet	5.627	4426.183	0.585	6.620	3827.632	0.832	2.847	1142.892	0.147
CRNet	8.053	4307.071	0.750	7.744	3982.434	0.868	2.482	1125.878	0.412
VCNet	7.620	4957.663	1.020	7.343	3636.134	1.043	**3.152**	**1362.968**	0.880
	± 0.052	± 32.858	± 0.007	± 0.011	± 31.744	± 0.008	± 0.005	± 3.406	± 0.001
CoLA	**8.931***	**5015.779***	**1.480***	**8.956***	**4104.366***	**1.409***	3.055	1327.621	**1.106***
	± 0.047	± 34.947	± 0.009	± 0.061	± 24.104	± 0.011	± 0.005	± 6.847	± 0.002

Table 2: Robust subsidy prediction accuracy within the true expected subsidy threshold right-side neighborhood.

	ML-1M, $K = 1$			ML-1M, $K = 2$		
γ	0.9	0.7	0.5	0.9	0.7	0.5
VCNet	0.275	0.278	0.281	0.433	0.427	0.415
OURS	**0.490**	**0.466**	**0.453**	**0.487**	**0.489**	**0.477**
	KuaiRec, $K = 0.5$			KuaiRec, $K = 1$		
γ	0.9	0.7	0.5	0.9	0.7	0.5
VCNet	0.340	0.231	0.229	0.408	0.452	0.439
OURS	**0.498**	**0.493**	**0.466**	**0.499**	**0.488**	**0.486**

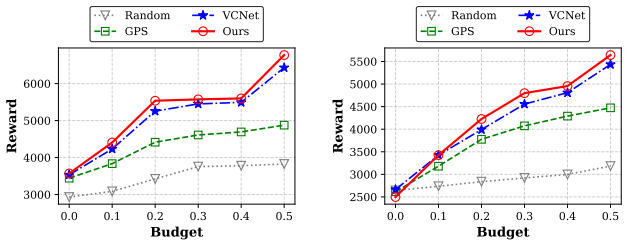
where we initialize $\beta = 0.2$. The term $\sum_{(u,i)\in O_{te}} R_{u,i}$ represents the total profit under the idealized scenario, where all test samples are purchased without any subsidy, i.e., $D_{u,i} = 0, \forall (u,i) \in O_{te}$. All methods are implemented using PyTorch, with the Adam optimizer employed for training. For all datasets, we tune embedding size in $\{16, 64\}$, batch size in $\{2048, 4096, 8192\}$, learning rate in $\{0.005, 0.01, 0.05\}$, and weight decay in $\{1e-6, 1e-5, 1e-4, 1e-3\}$.

Table 3: Results of the online A/B test.

	Day1	Day2	Day3	Day4	Day5
ILR	**+8.06%**	**+16.70%**	**+37.44%**	-16.00%	**+30.87%**
GTV	**+3.22%**	**+2.14%**	**+1.40%**	-3.97%	**+1.38%**
OR	-1.67%	**+1.16%**	**+1.27%**	**+1.14%**	**+1.62%**
	Day6	Day7	Day8	Day9	Day10
ILR	**+41.32%**	**+7.24%**	-0.69%	**+1.96%**	-1.57%
GTV	**+0.87%**	**+2.60%**	**+1.70%**	**+1.04%**	-2.22%
OR	**+1.84%**	**+0.09%**	-1.04%	-0.78%	**+4.66%**
	Day11	Day12	Day13	Day14	Day15
ILR	**+43.62%**	**+18.49%**	**+13.76%**	**+28.81%**	**+12.35%**
GTV	**+3.95%**	-0.53%	-1.40%	**+1.77%**	**+1.18%**
OR	**+1.00%**	**+3.79%**	**+5.71%**	**+1.09%**	**+1.43%**

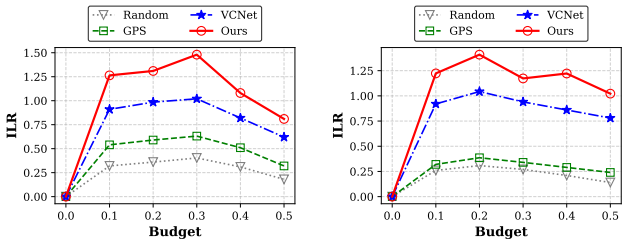
5.2 Performance Comparison

Table 1 reports the performance of all methods on three datasets under both unconstrained and budget-constrained settings. None method, which provides no subsidies, serves as the baseline for incremental leverage ratio (ILR) calculation. Random method allocates



(a) Reward on ML-1M.

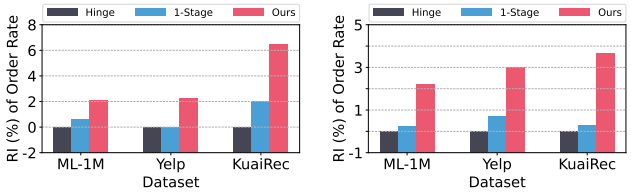
(b) Reward on YELP.



(c) ILR on ML-1M.

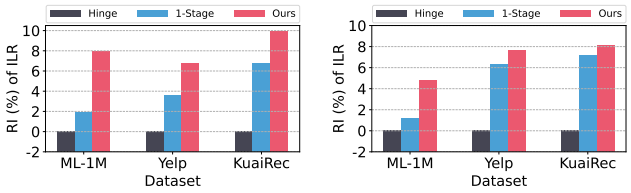
(d) ILR on YELP.

Figure 2: Effects of varying budget (tuned by hyperparameter β) on ML-1M and YELP datasets.



(a) OR with Budget.

(b) OR without Budget.



(c) ILR with Budget.

(d) ILR without Budget.

Figure 3: Effects of conditional dependence loss and asymmetric prediction loss across three datasets.

subsidies uniformly at random, yielding modest improvements in order rate and reward but low ILR due to inefficiency. S-Learner, which treats the intervention as a simple feature, consistently underestimates subsidies and performs worst, with costs remaining below budget. GPS, DRNet and CRNet show similar results, while GPS produces lower rewards and ILRs under comparable order rates, suggesting potential over-subsidization. VCNet outperforms DRNet on all metrics due to its flexible varying-coefficient prediction head, making it the strongest baseline. Our method achieves the best performance across datasets, with pronounced gains under budget constraints, confirming the effectiveness of the proposed two-stage learning framework with asymmetric loss.

Figure 2 examines the effect of budget size, controlled by hyperparameter β (larger β indicates higher budget). As budgets increase, all methods improve in reward, but ILR peaks at $\beta = 0.2$ and 0.3 and declines thereafter. Random performs worst due to indiscriminate allocation. GPS behaves similarly to Random at small budgets but improves with larger budgets while maintaining ILR better. VCNet is the most competitive baseline overall. Our method consistently outperforms all baselines, particularly under tight budgets, where it achieves a significantly higher ILR by accurately predicting subsidies just above expected values, enabling cost-efficient order completions and maximizing profits.

5.3 In-Depth Analysis

Table 2 evaluates whether the proposed robust subsidy prediction falls within the right neighborhood of the true expected subsidy threshold $D_{u,i}$, defined as the interval $[D_{u,i}, D_{u,i} + \sigma]$. We measure this as the probability that test samples' predictions fall within their respective true intervals, where σ corresponds to the sampling standard deviation of $D_{u,i}$ in the semi-synthetic experiments. In these experiments, the ground-truth threshold $D_{u,i}$ is influenced by hyperparameter K, while the actual subsidy $T_{u,i}$ in training data is affected by hyperparameter γ. Results across various hyperparameter settings show that our method consistently achieves a higher probability of predicting subsidies within the target interval, demonstrating its robustness.

5.4 Ablation Study

Figure 3 evaluates the effect of removing Stage 2 asymmetric loss and Stage 1 conditional independence loss. The full method is denoted as "Ours," the variant without Stage 2 loss as "1-Stage," and the variant removing both losses (trained solely with hinge loss) as "Hinge." Using "Hinge" as the baseline, we report relative improvements (RI).

Impact of Conditional Dependence Loss. A comparison between "1-Stage" and "Hinge" variants reveals that introducing the conditional independence loss L_{cd} does not significantly improve the Order Rate but leads to a notable increase in ILR and Reward. This indicates that while the hinge loss provides a reliable interval supervision, there are inevitable deviations from the true values. By incorporating the conditional independence constraint, the predictions more closely approximate the true expected subsidies.

Impact of Asymmetric Prediction Loss. A comparison between the "Ours" and "1-Stage" variants shows that incorporating the asymmetric loss $L_{\text{2-stage}}$ in the second stage substantially improves the Order Rate. This improvement stems from the asymmetric loss guiding the model to produce more robust subsidy predictions, specifically higher subsidies, leading to increased conversions. While the higher Order Rate contributes to greater overall rewards, it also results in relatively higher costs. Consequently, without budget constraints, the ILR improvement is less pronounced, whereas under budget constraints, the Sharpe ratio based high-efficiency allocation strategy mitigates this effect.

Table 4: Robustness analysis under violated conditional independence assumption. Best results are bolded. To align with real-world scenarios, the *OR* metric is reported in permille (10^{-3}).

With Budget	ML-1M			Yelp			KuaiRec		
Methods	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR
VCNet	4.538	**5195.723**	1.266	6.767	3649.383	1.028	2.724	**1351.307**	1.239
Ours	**5.308**	5048.316	**1.456**	**8.243**	**3946.012**	**1.434**	**2.734**	1324.388	**1.444**
Without Budget	ML-1M			Yelp			KuaiRec		
Methods	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR	OR ($\times 10^{-3}$)	Reward	ILR
VCNet	7.012	4741.640	0.919	6.711	3451.522	1.013	2.964	1235.567	0.854
Ours	**8.931**	**5015.77**	**1.480**	**8.956**	**4104.366**	**1.409**	**3.055**	**1327.621**	**1.106**

Table 5: Effects of hyper-parameter α with budget.

	ML-1M		Yelp		KuaiRec	
α	Reward	ILR	Reward	ILR	Reward	ILR
0	3729.996	1.128	3699.504	1.020	1284.238	1.338
1e-5	5191.716	1.391	4188.441	1.235	1360.912	1.422
1e-4	5243.452	1.505	4203.070	1.444	1446.626	1.416
1e-3	5486.290	1.571	**4226.012**	**1.454**	1447.836	1.432
1e-2	**5573.352**	**1.588**	3813.880	1.311	**1471.073**	**1.533**
1e-1	5339.879	1.436	3778.060	1.402	1187.237	1.334

Table 6: Effects of hyper-parameter α without budget.

	ML-1M		Yelp		KuaiRec	
α	Reward	ILR	Reward	ILR	Reward	ILR
0	3985.967	1.017	3088.032	1.123	1094.522	1.003
1e-5	4614.372	1.231	3833.403	1.327	1257.582	1.043
1e-4	4734.716	1.268	4003.370	1.348	1268.481	1.082
1e-3	4992.510	1.459	**4104.366**	**1.409**	1294.163	1.080
1e-2	**5015.779**	**1.480**	3781.571	1.379	**1327.621**	**1.106**
1e-1	4934.099	1.365	3375.989	1.240	1071.319	0.994

5.5 Online A/B Test

Table 3 presents the results of a 15-day online A/B test conducted on a real-world demand-side platform with over millions of daily active users. We compare our proposed method against the DRNet baseline across three key metrics: ILR, GTV, and OR. All improvements are reported as percentage gains relative to DRNet.

Our method demonstrates consistent and substantial improvements across the majority of test days. For ILR, we achieve positive gains on 11 out of 15 days and the average ILR improvement across all 15 days is +16.16%, indicating significantly higher subsidy efficiency. For GTV, our method shows positive gains on 11 out of 15 days, with an average improvement of +0.88%, demonstrating increased revenue generation. The OR metric exhibits positive improvements on 12 out of 15 days, with an average gain of +1.42%, confirming higher conversion rates.

Meanwhile, even on days with negative performance, the magnitude of decline is relatively modest compared to the substantial gains observed on other days. This pattern suggests that our method maintains robustness under varying real-world conditions while delivering consistent overall improvements. The consistency of gains across multiple metrics and the majority of test days validates the practical effectiveness of our two-stage robust optimal subsidy learning framework in production environments.

5.6 Robustness Analysis

Our estimation in Stage 1 utilizes the conditional independence assumption (i.e., $D \perp T \mid X$). However, in real-world recommendation platforms, this assumption may be violated. A common scenario is that the subsidy and the user's expectation mutually influence each other. For example, the subsidy T_1 allocated by the platform at a previous time influences the user's expected subsidy D_2 for the next purchase, and *at the same time*, the user's initial expectation D_1 influences the platform's next subsidy allocation T_2. To evaluate the robustness of our method in such scenarios, we constructed a setting that violates this assumption by simulating this two-way dependency. We define D_1 and T_1 as the user's initial expected subsidy and the platform's initial allocated subsidy, respectively. We then generate the new expectation D_2 and new subsidy T_2 using the following functions:

$$D_2 = D_1 \exp(\max\{T_1 - D_1, 0\}/10) \tag{11}$$

$$T_2 = T_1 \exp((D_1 - T_1)/10) \tag{12}$$

These functions create two outcomes based on the initial interaction:

- **If $T_1 > D_1$:** The user's expectation for the next purchase (D_2) **will increase**, as the max term is positive. Simultaneously, the platform's subsidy for the next allocation (T_2) **will decrease**, as the exponent $(D_1 - T_1)$ is negative.
- **If $D_1 \geq T_1$:** The user's next expectation (D_2) **will not change**, as the max term is zero. However, the platform's next subsidy allocation (T_2) **will increase**, as the exponent $(D_1 - T_1)$ is non-negative.

As shown in the Table 4, even when this assumption is violated, our method still comprehensively outperforms the VCNet baseline on the key metrics of profit (Reward) and subsidy efficiency (ILR). This demonstrates that our proposed two-stage framework possesses strong robustness.

5.7 Sensitivity Analysis

Tables 5 and 6 investigate the effect of the conditional independence loss (L_{cd}) coefficient α in Stage 1 on prediction performance, both with and without budget constraints. We assess Reward and ILR across three public datasets under varying α, with the best results highlighted in bold. The findings show that setting α within a moderate range effectively balances the hinge loss L_{hinge} and conditional dependence loss L_{cd}, leading to notable improvements in both metrics. Optimal performance is observed when α is set to $1e-3$ or $1e-2$. Conversely, excessively large values (e.g., $1e-1$) diminish the influence of L_{hinge}, thus negatively impacting subsidy prediction accuracy.

6 Conclusion

This paper investigates optimal subsidy policy learning in marketing. We highlight that predicting the expected subsidy threshold is distinct from treatment effect estimation. To bridge this gap, we propose CoLA, a two-stage robust optimal subsidy learning method. Specifically, the first stage generates a coarse estimate using factual subsidy data and conditional independence assumptions; the second stage refines it via an asymmetric loss function to introduce a robust upward bias. Experiments on three public datasets and an online A/B test show our method consistently maximizes total profit and incremental leverage.

Despite its effectiveness, a limitation of this work is that it primarily focuses on profit maximization within a single transaction, thereby treating subsidy allocation as a static decision process. However, marketing subsidies often exert long-term cumulative effects. Frequent or excessive incentives may lead to "subsidy dependency" among users, potentially diminishing their long-term brand loyalty or perceived value. Consequently, our current framework does not yet incorporate the long-term modeling of user Lifetime Value.

Acknowledgment

Z. Lin was supported by the NSF China (No. 62276004) and the State Key Laboratory of General Artificial Intelligence.

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